

The following are some notes I prepared (in 2008) to help me clarify a leading theme in Badiou's philosophy. My need to sort out some issues in Badiou's philosophy was to help me triangulate ideas on harmony Leibniz's philosophy, Badiou's ontology, and Hollis Frampton's ideas on harmony. These reflections will appear in a long essay in volume on Frampton by wife, Kathryn Elder. But discussions with colleagues have led me to conclude that the find Badiou's ontology befuddling, and I hoped these notes might provide them with a way in. But these are just notes—for example the format of citations is inconsistent.

Alain Badiou's philosophy is philosophy in the old style: he has proposed a system. He has strived to develop a system whose propositions would be free of equivocation, would be precise, and, more extraordinary yet, demonstrable. In his effort to achieve this goal, he has turned to mathematics, believing that only mathematics has sufficient legitimacy to serve as the foundation—or better, the armature—of his system. One reason he embraces mathematics is that mathematics dismantles what he considers the perilous theological connection Truth-Being-One—indeed, one respect in which Badiou's philosophy differs from most traditional philosophical systems is that he rejects that connection.

The core conviction on which Badiou founds his system (and his philosophy is very systematic) is, as he puts it “ontology = mathematics.” Badiou explains the reason he believes this by pointing out that being is fundamentally pure multiplicity, including infinite chains of multiplicities and that the most formalized, most complete framework of axioms of the multiple today is set theory. According, he develops his system from the axioms of set theory. He explores the axioms of set theory, taking them as statements about being. The axiom of extensionality can be used to give an idea of his approach. That axiom states that two sets that have the same elements are identical. Now, the idea of identity is philosophical fraught: notions such as identity and difference, same and other are fundamental to ontology. If one accepts that the being is multiplicity, then the axiom of extensionality has implications for how ontology should think of identity and difference. And as it is with the axiom of extensionality, so it with all the axioms of set theory: they constitute, Badiou believes, a coherent body of propositions about being qua being (provided one accept, as Badiou does, the proposition that being is reducible to pure multiplicity). We could see this from another point of view: Leibniz defined harmony (which, as we saw above, is analogous to transfinite set theory), as “diversity compensated by identity”; as “unity in multiplicity” ; as “unity in variety” ; as “simplicity in multiplicity”; and as “unity in plurality. The theme of all these characterizations is the multiple become one. The multiple becoming one is the fundamental process of set theory. But the issue of the one and the many is a topic with which any ontology must deal.

In an interview with Lauren Sedofsky, published in *Artforum* in October, 1994, Badiou asserted “Ultimately, being qua being is nothing but the multiple as such. What there is is the multiple. Mathematics is the kind of thought, and consequently the kind of discourse, that apprehends the configurations of multiplicity independently of any characteristic other than their multiplicity. From the moment that what is being taken into account is being qua being, that is, pure multiplicity, it is indispensable that language rid itself of its equivocalness. (There we are with Lacan.) Formalization, this way of tearing language away from its status of mother tongue, this transformation of the mother tongue into a

tongue that no longer offers any natural reception for the speaker, is the discipline through which thought appropriates the form of the pure multiple.

One of the central ideas of Badiou's philosophy is that of the situation. A situation, essentially, is what we find ourselves in—it is, of course, a manifold of manifolds. The situation is a multiple, a multiple that is infinite because all situations in reality are infinite. Situations typically possess a number of 'states'; according to Badiou's own description, a particular state of a particular situation is constituted by the order of its subsets. Every situation has a number of languages associated with it, at least one language for every state. The language associated with a particular state of a situation aims at showing how an element belongs to such and such a subset.

The glossary to *L'être et l'événement* (Being and Event) defines ontology as: "Science of being-qua-being. Presentation of presentation. Realized as thought of the pure multiple, thus as Cantorian mathematics or set theory" (BE 517/551). Badiou describes a situation as that which presents the elements that constitute it and the state of the situation is what presents, not the situation's elements, but its subsets. Badiou's description of the situation harkens back to Leibniz's notion of a ("*die Einigkeit in der Vielheit*, or unity in multiplicity). Badiou points out that every identifiable being is always already in situation. This means that every being is a consistent multiplicity, a "counted-for-one." What is not in situation, what is not this thing or that thing (and so "counted-for-one"), can only be qualified as no-thing. Consequently we may say that "there are" only situations, that is, consistent one-multiples.

But this, Badiou points out, leads to an impasse for ontology (and indeed for thought). For if the multiple is, then being is not equivalent to the one. And yet there is a presentation of this multiple only if what is presented is one. Or again, if being is one, then the multiple cannot be; on the other hand, if presentation is multiple, then the multiple must be. Badiou proposes that this deadlock can only be broken only by deciding that the one, strictly speaking, is not – that the one can only be a *result*, a presented multiplicity which has been "*counted for one*." A consistent multiplicity of this sort Badiou calls a situation, and every situation must have a structure which lays down the principle according to which it is count-for-one (that is, the situation is a set, for which there must be condition, or principle, defining what belongs to the set). (BE 23–24/31–32).

Another key notion in Badiou's system is that of an event. An event has the power to puncture a situation. The event is situated in the situation, it has a site in it; but it does, only a retroactive temporality: the temporality of after-the-event), but it leaves traces, traces that allow an encounter (*rencontre*) for elements of the situation.

Badiou characterizes his system as a 'Platonism of the manifold.' There is more than a little impudence in this—Plato, after all, is the philosopher of the One. Moreover, as one might have inferred from his curt dismissal of what he deems the perilous theological connection of Truth-Being-One, Badiou, like Deleuze, is a radical immanentist. Anything that smacks of transcendence for him is anathema. Nonetheless, there are grounds for Badiou's characterization of his system. To understand them (and understanding them is a key to understanding the relevance of Badiou's ideas to Hollis Frampton's aesthetic theory and artistic practice), we must first consider Badiou's idea of truth.

For, knowledge is what language represents of states of events (that is, of the order of the subsets in an infinite set known as an event). Badiou, a truth is what, in a situation, knowledge cannot see, what its language cannot utter: a truth is a puncturing of such knowledge. As a result, it can have no truck with meaning either: a truth, Badiou claims,

is that which doesn't make sense in a situation. As 'a hole in meaning' truth cannot be the object of a hermeneutic procedure.

A truth, then, is 'outside meaning' Badiou sets out four 'modalities' of truth: it is undecidable, for it is linked to the aleatory advent of an event; it is indiscernible—the process of truth is not governed by any internal or external necessity; it is generic—a truth has no characteristic expressible within knowledge; and it is unnameable—forcing a truth by naming it within the language of the encyclopaedia (the ensemble of all knowledge) destroys it, the only name of a truth must be outside the language of the situation. The task of philosophy is to follow this negative path, to extract, or rather subtract the truth from knowledge and meaning. A truth, therefore, is always truth in a situation, but it is not of the situation: it is a puncturing of the knowledge that accounts for the situation, a process triggered by an emergence, within the given, of the radically new—an event.

Badiou's notion of event is connected to his notion of a special sort of situation that he calls an "ontological situation." An ontological situation is the situation that offers an account of the "one" of situational being in general. This "one" is not the One of Plato (and even less, the One of Plotinus).

As Badiou says, the ontological situation, which is here to be understood as set theory, "presents presentation."¹ That is to say, it counts as "one" what is presupposed by and counted in every situation: inconsistent multiplicity, formalized within set-theory ontology as the void set. And this is so even if, at the limit, this "one" must be thought of as a universal "for all" or an open "generic multiple" as opposed to a finished totality.¹

Badiou draws on work by the great American mathematician, Paul Cohen, to work out his idea of an inconsistent multiplicity. What Badiou takes from Cohen is absolutely crucial to his (Badiou's) system and will turn out to be key to Hollis Frampton's thoughts on unification in multiplicity. And to grasp Cohen's work on what is known as the continuum hypothesis (at least securely enough to allow us appreciate what Badiou has drawn from it) we have to have some understanding of the historical evolution of that problem which Cohen addressed. (The continuum hypothesis is the conjecture that the cardinality of \aleph_1 , second denumerable infinity is the same as the cardinality of the powerset of the first infinity [that is $\aleph_1 = 2^{\aleph_0}$].)

Cantor's continuum hypothesis, and the set theoretical method on which it was based, led to the insight that from the first, denumerably infinite set, we can generate an infinite sequence of every larger infinite numbers – an infinity of distinct infinities. We do so by applying the powerset operator to any term in the series in order to generate the next term. From this we get $\aleph_0, 2^{\aleph_0}, 2^{2^{\aleph_0}}, 2^{2^{2^{\aleph_0}}}$ Cantor had showed that the (infinite) value he called c , the number of real numbers, is equal to 2^{\aleph_0} . Cantor wondered whether the number of real numbers might be equal to \aleph_1 , the next infinite number after \aleph_0 . He was convinced that it was, and this conjecture came to known as the "continuum hypothesis."

Cantor's conjecture might seem recondite—to be the sort of thing to which philosophers need not pay attention. That impression is decidedly incorrect. For what turns on this conjecture is the relation between mathematical structures and the physical world of continuous space and continuous time. If Cantor's continuum hypothesis is true, then there is a secure relation between the structures for transfinite set theory and the

real numbers used in physical science. What is more, there would be a well-ordered relation between the hierarchy on infinite cardinalities and the real numbers. So, if Cantor's continuum hypothesis is true, then there would be a secure link not only between physical continuity (the continuity of space and time) but also between the physical continuum and the transfinite realm (for, then, by repeated application of the continuum theorem, as it then would be, there would be an exact equivalence between the transfinite numerical sequence, \aleph_0 , \aleph_1 , \aleph_2 , \aleph_3 , and the powerset series \aleph_0 , 2^{\aleph_0} , $2^{2^{\aleph_0}}$, $2^{2^{2^{\aleph_0}}}$. The universe in which Cantor's continuum hypothesis held true ($\aleph_1 = 2^{\aleph_0}$) would be the smallest transfinite universe possible—so we would know \aleph_1 's place in the hierarchy of transfinite numbers. However, if the continuum hypothesis proved either wrong, or incapable of being proved then we could not establish 2^{\aleph_0} 's place in the hierarchy of transfinite numbers. A mathematical universe that denied Cantor's continuum hypothesis would be inflicted with a baleful uncertainty: it would accept the existence of sets without knowing how to rank them in relation to other sets. We could think of the smallest transfinite power set (2^{\aleph_0}) exceeding \aleph_0 by an immeasurable gap.

Cantor was unable to prove his continuum hypothesis. In 1963, Paul Cohen was able to demonstrate that the continuum hypothesis is independent of the basic axioms of set theory. That is to say, the continuum hypothesis could be false while the axioms are true; or, the axioms of set theory do not entail the continuum hypothesis. Cohen revealed the continuum hypothesis to be an absolutely undecidable statement, i.e., a set theoretical statement that is well-formed, but which cannot be proved from the axioms of set theory. In demonstrating that the continuum hypothesis is independent of the axioms of set theory, Cohen exposed a gap between what mathematics, the science of multiplicity, can tell us about being and pure being itself. Issues around numbering 2^{\aleph_0} (that is, of locating it within the series \aleph_1 , \aleph_2 , \aleph_3 , . . . , \aleph_{17} . . .) or, what is the same thing, of measuring the continuum, are an impasse for the science of multiplicity. Nonetheless, this fissure in the science of multiplicity is opened up within the science of multiplicity itself. Badiou sometimes speaks of this in Lacanian terms, saying that the impossibility of measuring the numerical continuum is the Real of mathematical measurement itself.

Easton was able to extend Cohen's work, by showing that while 2^{\aleph_0} might equal \aleph_1 (that is, he was able to show that there is no inconsistency arises from asserting that 2^{\aleph_0} equals \aleph_1) 2^{\aleph_0} might also equal \aleph_{17} , or any \aleph_n —(that is, no inconsistency arises from positing that 2^{\aleph_0} equals \aleph_n). We could put Easton's finding in another way, as saying none of these forms of the continuum hypothesis can be deduced from the axioms of set theory, and so which form we choose is simply a matter of arbitrary choice. Badiou asserts, "we must tolerate the almost arbitrary situation of choice. . . . That [the concept of] quantity, this paradigm of objectivity, leads to pure subjectivity, this is what I would happily call the Cantor-Gödel-Cohen-Easton symptom" (*L'être et l'événement*,

309).

In respect to the revelation of the non-objectivity of what is multiple and the affirmation of the subject, the structure of this event is typical of the structure of events in general: truth, Badiou asserts, has no need of an object precisely because it is the result of an infinite procedure or generic multiple. An undecidable event occurs that supplements a multiple-situation and leads to the production, within multiple-being itself, of the truth of this event-supplemented situation as a generic multiplicity. Finally, it must be said that while there is no object, there is indeed a subject, which is itself a finite moment of this infinite generic procedure (v. Alain Badiou, *Manifesto for Philosophy*. Trans. Norman Madarasz. (Albany: State University of New York Press, 1999.) 95–96/77–78, 108/91). For Badiou, Cohen's proof, within what mathematicians call Zermelo-Fraenkel with Choice, (ZFC, an axiomatic system for set theory in which the basic Zermelo-Fraenkel axiom system is supplemented by the Axiom of Choice), of the essential inconsistency of multiple-being, reproduces within set theory, and so within ontology, the generic or "one"-truth of inconsistent multiple-being in general.

We are getting ahead ourselves here, to make this point, but the idea the axioms of set theory lead to an completely arbitrary choice, and the understanding we shall soon possess, that this arbitrary choice relates to the idea of generic being itself, might bring readers to recall that Frampton proposed that there was no solution to the Knight's Tour, or that it was infinite game, because it was impossible to determine where the tour should begin – that, we recall, is the only way of making sense of Frampton's claim that there is no solution to the Knight's Tour, since there is, in fact, a well-known algorithmic solution to the Knight's Tour of any chessboard of dimension $\mathbf{N} \times \mathbf{N}$ (one known to almost every programmer, since developing a solution to that problems, as well as to the famous Tower of Hanoi problem, is ordinarily given to computer science students, to introduce them to the concept of recursion). Let us suppose that the problem with beginning the Knight's Tour, Frampton's circumnavigation of all possible types of knowledge – indeed, his itinerary through what Badiou calls the encyclopaedia – is the problem of identifying the ground of all knowledge, that is, according to traditional ontology, knowledge of the One. We would have to begin with knowledge of generic being, being before it is distinguished into knowledge of particulars (or, in Badiou's terms, knowledge of multiplicity). A key implication of Cantor's transfinite set theory is that the finite must be understood in terms of the infinite: it is only as a consequence of the decision to assert the existence of an infinite number that we can characterize a certain sequence of numbers as a finite sequence, as something left behind in the limitation of the infinite. (*L'être et l'événement*.p 179) But the idea that the finite must be understood in terms of the infinite is also what Grossteste proposes in "On Form, or The Ingression of Light," parts of which Frampton translated and used in the third part of the films. We shall see that Badiou argues that impossibility of understanding generic being is implied by the axioms of set theory.

Badiou tells us, it was this discovery which led him to identify mathematics and ontology. (*L'être et l'événement*. 11). Badiou considers Cohen's discovery as one of the monumental intellectual events of recent times and the most decisive influence on the composition of *L'être et l'événement*. He writes that "Cohen's theorem completes . . . the modernity opened up by the distinction between thought and knowledge." (C 203). Axiomatic set theory, which was constructed partly to provide a foundation for Cantor's transfinite number scale cannot show us how to put these ideas together with the set theoretical ideas related to simple counting procedures. Badiou uses this idea of the generic, unqualified one to establish that not only can set theory be an ontology, but that

it is the only ontology. Only what is in situation truly “is,” and since this unqualified being is not in situation it “is” not. Nonetheless, it is presented, and since being is presented in every presentation, it has being, at least in the sense of being a presented presentation. The structure of this inconsistent or unqualified being qua being must be a situation capable of presenting inconsistent multiplicity as that from which every in-situation “thing” is composed. It will “present presentation” in general (BE 27–28/35–36). The very structure science of this “present presentation” must have a way of showing that this no-thing exists, and that everything in the ontological situation is composed out of it, and must do without assigning to it any other predicate other than its multiplicity. Only the axioms of set theory, fulfil this requirement, Badiou avers, for they only give an implicit definition of what it operates on: the pure multiple (v. BE 28–30/36–38, 52–59/65–72). Since set theory alone meets these requirements regarding unqualified being, it alone can serve as ontology.

Set theory meets the requirement of the ontology of the “presented presentation” by reducing the one to the status of a relationship, the relationship of simple belonging (written \in). In doing so, it eliminates the role of the concept from the presented presentations ontology, for everything will be presented, not according to the one of a concept, but only according to its relation of belonging or counting-for-one. Apart from equality ($=$), which, for set theory, as we have seen, is defined in terms of sets’ extension, the relationship of set membership, or belonging, is the only fundamental relationship in set theory. Whether or not an object belongs to a set provides the foundation for the entire lexicon of mathematics, and so ontology. To put this another way, to say that ‘something = α ’ is to say that that thing possess attribute α or, what is the same, that it belongs to the set of things, β , with attribute α . Badiou’s way of saying this is that only be presented according to a multiple β – this idea of being presented according to multiple β is what is written as $\alpha \in \beta$ (or ‘ α is an element of β ’).

A second character of set theory that enables it to serve as the ontology of the “presented presentation” is that set theory does not distinguish between “objects” and “groups of objects,” or between “elements” and “sets.” The variables of set theory are of a single type. This, entails, amongst other things, that being an element of a is not an intrinsic quality of the elements of the set. Rather, it is a simple relation, ‘ α is in a relation to β , the relation of “being-an-element-of” – α could have any number of intrinsic properties (for example, it could be an object or a group of objects). Thus, by the uniformity of its variables, sets, and so set theory, can indicate that it does not speak of ‘the one,’ and can do so without defining either ‘the one’ or the elements of the set. All that set theory presents – presents by its very rules – are multiples of multiples: multiples belonging to or presented by other multiples.

Badiou sometimes expresses this by saying that set theory is a ‘subtractive ontology’ – it subtracts the qualities of the elements with which it deals, and treats them only as members of a set. It does not try to identify the objects of the world on the grounds of their inherent attributes – even less does it try to carve up the world into types defined by the properties they possess.

A third feature of the set theory that enables it to serve as ontology derives from the “axiom of separation.” The axiom of separation states that, given a set α , one can separate a subset β that make the formula F true. Let the set α be the set of all apples and let formula F be the formula, “ x is rotten.” The axiom of separation states that we can separate a subset β from α that makes the formula “ x is rotten” true whenever a member of β is substituted for ‘ x .’ The inference Badiou draws from this is that language causes a split or division in existence. From that Badiou concludes that being

must be in excess of language. For Badiou, this implies a meta-ontological implication of the axiom of separation is that an undefined (generic) existence must always be assumed in any sort of definition. Badiou arrives at this conclusion by recognizing that, though set theory may, through the “axiom of separation,” affirm property or formula of language, it does not directly present a collection with that property as an existing multiple. Rather, the presentation of a collection with that property can only be a “separation” or sub-set of an already presented multiplicity. That is to say, a property only determines a multiple under the supposition that there is already a presented multiple (BE 43–48/54–59). Everything thus hinges on the determination of the initial pure multiple. But we saw above that this initial, pure multiple must be an inconsistent multiplicity, This inconsistent multiple, the unrepresentable of presentative consistency, is the absolutely initial multiple from which other separations are taken. This unrepresentable of presentative consistency is the excess of being over language.

How does set theory assure the existence of this absolutely initial multiple, this void, so that all ontology’s (i.e., set theory’s) compositions can be taken (separated) from it. Badiou’s answer is startling: set theory does so, As Badiou says, by making this nothing be by giving it a pure proper name: \emptyset (BE 66–67/80). That the void is presented through that name is not to say that the void is thereby one. What is named is not the one of the void, but rather its uniqueness, its “unicity.” In what sense is the void unique? Badiou states that the axiom of extensionality explains sense, for the axiom of extensionality fixes the rule of the difference or sameness for any two multiples whatsoever. However, the void, that is, the null set, has no elements; hence there can have no conceivable differentiating mark for this set (that is no difference between the set of elements it contains and null set that is a subset of any other set). That established there is a ‘unicity’ of the unrepresentable within presentation – that is, there cannot be “several” voids: the void is unique and this is what is signalled by the proper name, \emptyset (BE 67–69/80–83).

And now, a few words on Hollis Frampton. He states in “A Lecture” So we have been here before, . . . No not in this very room but in this generic darkness . . .” He even gives an account of this notion of a generic space by deploying a Badiouian subtractive process: “We are, shall we say, comfortably seated. We may remove our shoes, if that will help us remove our bodies. . . . So we are suspended in a null space.” Or, again, in his dialogues with Carl Andre, he distinguishes between beings that have qualities (all that is) and the theoretical unqualified being. Unqualified beings are atoms, he suggests; he expresses this by saying that “atoms are null cases”

I intend to follow up with commentary on Frampton’s thoughts on harmony.

- 1 V. Alain Badiou, *Being and Event*. Trans. Oliver Feltham. London and New York: Continuum, 2005, 27; originally published in French as *L'être et l'événement*. Paris: Éditions du Seuil, 1988, 35–36.
- 1 See in particular on this: Alain Badiou, “Eight Theses on the Universal” in Badiou: *Theoretical Writings*. Ed. Ray Brassier and Alberto Toscano. London and New York: Continuum, 2006, 143–152. Originally published as “Huit thèses sur l’universel” in Badiou, *Universel, singulier, sujet*. Ed. Jelica Sumic. Paris: Kimé, 2000, 11–20.